

THE SECRET MACHINE LAB

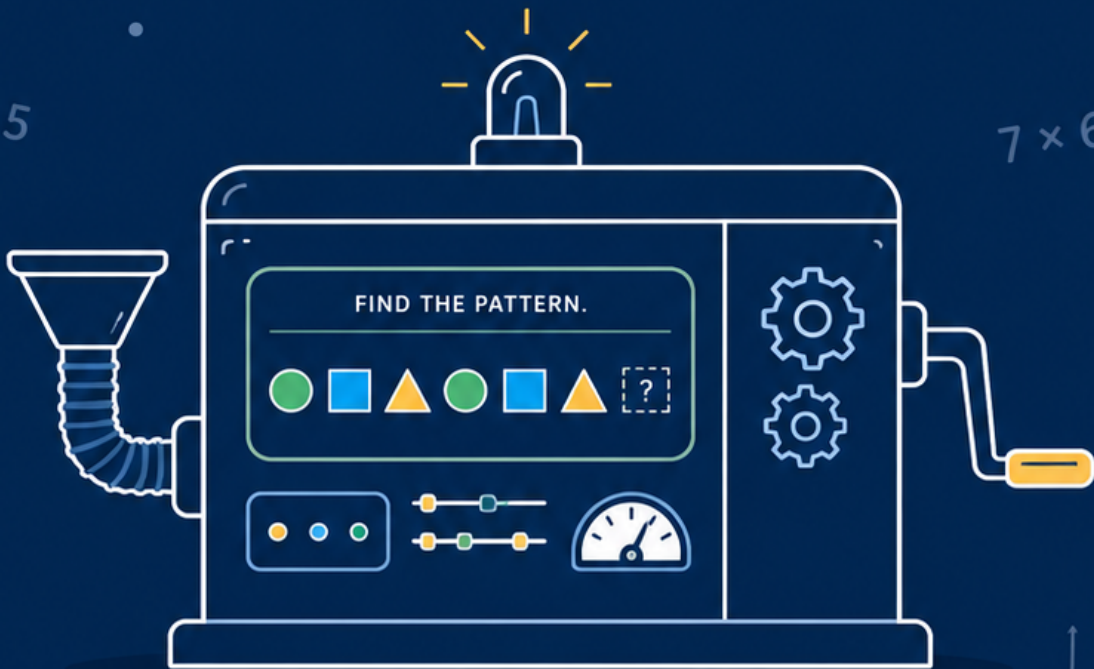


A Pattern Detective Workbook for 4th Grade

$$2 + 3 = 5$$

$$7 \times 6 = 42$$

$$\frac{3}{4}$$



THINK. SPOT. SOLVE.

FOURTH
GRADE

NAME

Welcome to the Secret Machine Lab

You are not here to memorize rules. You are here to discover them.

Your job as a Pattern Detective

A secret machine takes an **input**, follows a hidden rule, and produces an **output**. You get clues about what happened. Your job is to figure out what must be going on inside.

For example:

$$2 \rightarrow 7, \quad 3 \rightarrow 10, \quad 4 \rightarrow 13$$

One simple rule that fits every clue is: **multiply by 3, then add 1**. In symbols, a grown-up might write $3x + 1$. You do not need that notation yet. The important part is the thinking.

The Pattern Detective Cycle

1. **Notice.** What changes? What stays the same?
2. **Guess.** Invent a rule that might explain the clues.
3. **Test.** Does your rule work for every clue?
4. **Explain.** Say the rule clearly enough that someone else could use it.
5. **Challenge.** Could another rule fit? What new clue would help?

A note for grown-ups

These puzzles are deliberately different from ordinary “find the next number” worksheets. The child is practicing **inference from evidence**: forming a hypothesis, checking it, working backward, comparing explanations, and sometimes noticing that the evidence is not sufficient for a unique answer. That is algebraic thinking before the algebra vocabulary arrives.

Name:

Lab partner (optional):

Detective Toolkit

Four habits that make hard puzzles easier

Tool 1: Look at the change

If $5 \rightarrow 12$, ask: "How could I get from 5 to 12?" Maybe $+7$. Maybe $\times 2 + 2$. One clue can allow many ideas. More clues give you more evidence.

Tool 2: Test every clue

A rule is not good just because it fits one pair. If you think the rule is $+5$, then $3 \rightarrow 8$ fits. But if another clue is $6 \rightarrow 18$, your rule fails. Back to the evidence!

Tool 3: Work backward

If the machine says " $\times 4$, then $+2$ " and the output is 30, undo the steps in reverse order: $30 - 2 = 28$, then $28 \div 4 = 7$.

Tool 4: Say exactly what you mean

"It gets bigger" is an observation. "Multiply by 3 and add 1" is a rule someone can actually use.

Warm-up thought experiment

You see only one clue: $4 \rightarrow 12$. Write **three different rules** that could explain it.

Lab 1: One-Step Machines

Start with rules you can see in one move

Mission

For each machine, write the hidden rule and fill in the missing output.

1

$$2 \rightarrow 7$$

$$5 \rightarrow 10$$

$$9 \rightarrow 14$$

$$12 \rightarrow$$

Rule:

3

$$20 \rightarrow 13$$

$$16 \rightarrow 9$$

$$12 \rightarrow 5$$

$$25 \rightarrow$$

Rule:

2

$$4 \rightarrow 12$$

$$6 \rightarrow 18$$

$$9 \rightarrow 27$$

$$11 \rightarrow$$

Rule:

4

$$6 \rightarrow 3$$

$$14 \rightarrow 7$$

$$22 \rightarrow 11$$

$$30 \rightarrow$$

Rule:

5

$3 \rightarrow 15$

$7 \rightarrow 35$

$10 \rightarrow 50$

$12 \rightarrow$

Rule:

6

$24 \rightarrow 6$

$36 \rightarrow 9$

$44 \rightarrow 11$

$60 \rightarrow$

Rule:

Detective note

Which clues were easiest: addition/subtraction or multiplication/division? Why?

Lab 1B: Rule or Impostor?

Choose the rule that fits ALL the evidence

7 $1 \rightarrow 6, 4 \rightarrow 9, 10 \rightarrow 15$ A. $+5$ B. $\times 5$ C. $+6$

Answer:

Reason:

10 $12 \rightarrow 3, 28 \rightarrow 7, 40 \rightarrow 10$ A. $\div 4$ B. -9 C. $\times 4$

Answer:

Reason:

8 $2 \rightarrow 8, 5 \rightarrow 20, 9 \rightarrow 36$ A. $+6$ B. $\times 4$ C. $\times 3$

Answer:

Reason:

11 $4 \rightarrow 24, 7 \rightarrow 42, 11 \rightarrow 66$ A. $+20$ B. $\times 6$ C. $\times 5 + 4$

Answer:

Reason:

9 $18 \rightarrow 15, 10 \rightarrow 7, 25 \rightarrow 22$ A. -3 B. $\div 3$ C. $+3$

Answer:

Reason:

12 $30 \rightarrow 20, 18 \rightarrow 8, 13 \rightarrow 3$ A. -10 B. $\div 2 + 5$ C. $+10$

Answer:

Reason:

Explain one

Pick any puzzle above. Explain why the other two rules fail.

Puzzle number:

Explanation:

Lab 2: Two-Step Machines

Now the machine has two gears inside

Mission

Find the two-step rule. The examples are chosen so a simple rule using multiplication or division plus addition or subtraction works cleanly.

13

$1 \rightarrow 4$

$2 \rightarrow 7$

$5 \rightarrow 16$

$8 \rightarrow$

Rule:

15

$3 \rightarrow 13$

$5 \rightarrow 21$

$8 \rightarrow 33$

$10 \rightarrow$

Rule:

14

$2 \rightarrow 9$

$4 \rightarrow 17$

$6 \rightarrow 25$

$10 \rightarrow$

Rule:

16

$4 \rightarrow 10$

$6 \rightarrow 14$

$9 \rightarrow 20$

$12 \rightarrow$

Rule:

17

$5 \rightarrow 13$

$10 \rightarrow 23$

$20 \rightarrow 43$

$30 \rightarrow$

Rule:

18

$6 \rightarrow 16$

$9 \rightarrow 25$

$12 \rightarrow 34$

$20 \rightarrow$

Rule:

Lab 2B: Order Matters

Same two operations. Different order. Different machine.

Example

Start with 3.

Machine A: $\times 2$, then $+5$: $3 \times 2 + 5 = 11$.

Machine B: $+5$, then $\times 2$: $(3 + 5) \times 2 = 16$.

The operations are the same, but the order changes the answer.

19. Which machine made these clues?

$1 \rightarrow 8$, $2 \rightarrow 10$, $5 \rightarrow 16$

A. $\times 2$, then $+6$ B. $+6$, then $\times 2$

Answer: Test it on input 9: output =

20. Which machine made these clues?

$2 \rightarrow 15$, $4 \rightarrow 21$, $7 \rightarrow 30$

A. $+3$, then $\times 3$ B. $\times 3$, then $+9$

Trick question?

What do you notice?

21. Compare

For input 4, find both outputs.

Machine P: $\times 5$, then -2 :

Machine Q: -2 , then $\times 5$:

Difference between the outputs:

22. Make order matter

Choose two operations and an input. Show that switching the order changes the answer.

Lab 3: Reverse Engineering

The output is known. Find the input.

Mission

Undo the machine in reverse order. Show enough work that another detective can follow your thinking.

23

Rule: $\times 3$, then $+1$

$\square \rightarrow 22$

Input =

Work:

25

Rule: $\times 5$, then $+7$

$\square \rightarrow 47$

Input =

Work:

24

Rule: $\times 4$, then -3

$\square \rightarrow 29$

Input =

Work:

26

Rule: $\times 2$, then $+9$

$\square \rightarrow 35$

Input =

Work:

27Rule: $\div 3$, then $+4$ $\square \rightarrow 12$

Input =

Work:

28Rule: $\div 4$, then -2 $\square \rightarrow 7$

Input =

Work:

Say the strategy

If a machine says “multiply by 6, then add 8,” what should you do first when working backward?

Lab 3B: Inputs, Outputs, and Missing Clues

Sometimes the mystery can be anywhere

Problem	Rule	Input	Output
29	$\times 3, +2$	6	
30	$\times 4, -1$		31
31	$+9$	14	
32	$\div 5$		7
33	$\times 2, +6$	12	
34	$\times 5, -4$		41
35	-13	50	
36	$\div 4, +3$		8

37. Missing rule

A machine gives these clues: $3 \rightarrow 11$, $5 \rightarrow 17$, $8 \rightarrow 26$.

Rule: _____ What is $12 \rightarrow$?

38. Missing clue

A machine follows $\times 4, +3$. Invent an input-output clue that proves the machine is *not* simply "+6."

Your clue: _____ $\rightarrow$ _____

Why it helps: _____

Lab 4: Broken Machines

One clue is wrong. Find the bad evidence.

Mission

Each box is supposed to use one simple rule. Usually exactly one input-output pair is incorrect. Circle the broken clue, then write the rule that fits the others. Stay alert: once in a while, the real mystery may be the directions themselves.

39

$2 \rightarrow 9$
 $4 \rightarrow 15$
 $6 \rightarrow 20$
 $8 \rightarrow 27$
Broken:

Rule:

41

$4 \rightarrow 18$
 $6 \rightarrow 26$
 $8 \rightarrow 34$
 $10 \rightarrow 40$
Broken:

Rule:

40

$3 \rightarrow 14$
 $5 \rightarrow 22$
 $7 \rightarrow 30$
 $9 \rightarrow 35$
Broken:

Rule:

42

$5 \rightarrow 12$
 $10 \rightarrow 22$
 $15 \rightarrow 32$
 $20 \rightarrow 45$
Broken:

Rule:

43

$$8 \rightarrow 5$$

$$12 \rightarrow 7$$

$$16 \rightarrow 9$$

$$20 \rightarrow 11$$

Broken:

Rule:

44

$$6 \rightarrow 16$$

$$9 \rightarrow 25$$

$$12 \rightarrow 34$$

$$15 \rightarrow 44$$

Broken:

Rule:

Lab 4B: Repair Shop

Fix the clue instead of throwing it away

45

Supposed rule: $\times 5, +2$

Clues: $2 \rightarrow 12$, $4 \rightarrow 22$, $6 \rightarrow 31$, $8 \rightarrow 42$

Which clue is broken?

Repair it: $\rightarrow$

46

Supposed rule: $\times 2, -3$

Clues: $5 \rightarrow 7$, $8 \rightarrow 13$, $10 \rightarrow 17$, $12 \rightarrow 20$

Which clue is broken?

Repair it: $\rightarrow$

47

Supposed rule: $\div 3, +4$

Clues: $9 \rightarrow 7$, $15 \rightarrow 9$, $21 \rightarrow 11$, $30 \rightarrow 15$

Which clue is broken?

Repair it: $\rightarrow$

48. The suspicious scientist

A scientist claims the rule is " $\times 4, +1$." She shows only $2 \rightarrow 9$. Is that enough evidence to prove her rule? Explain.

Lab 5: Not Enough Evidence!

Sometimes several rules fit the same clue

Important mathematical idea

If you know only that $4 \rightarrow 12$, there is no single guaranteed rule. " $\times 3$ " works. So does "+8." So does " $\times 2, +4$." A good detective asks for **more evidence**.

49. One clue, three stories

Clue: $5 \rightarrow 20$. Write three different rules that fit.

Rule A:

Rule B:

Rule C:

50. A new clue arrives

You know $5 \rightarrow 20$. Two suspects remain:

Rule A: $\times 4$ Rule B: $\times 3, +5$

What single additional input would you test to tell the rules apart? Input:

What outputs would the two rules predict? A: B:

51. Same clue, different machines

Both machines send $3 \rightarrow 10$.

Machine P: $\times 3, +1$ Machine Q: $\times 2, +4$

Find an input where their outputs are different. Input:

P output: Q output:

Lab 5B: Ask for the Best Clue

Design an experiment to identify the machine

Mission

You may choose an input and ask the mystery machine for its output. Pick a useful input, predict what each suspect rule would do, then decide what the answer would tell you.

52

Suspect rules: A. $\times 2 + 3$ B. $\times 3 + 1$

Choose input: A predicts: B predicts:

If the machine returns your A prediction, what have you learned?

53

Suspect rules: A. $+10$ B. $\times 2$

You foolishly test input 10. What happens? A predicts: B predicts:

Why was input 10 a bad experiment?

Choose a better input: A predicts: B predicts:

54. Detective strategy

What makes an input a **good test** when you are choosing between two possible rules?

Lab 6: Machine Chains

One machine can feed another

Example

Machine A adds 4. Machine B multiplies by 3.

Input 2 goes through A: $2 + 4 = 6$. Then through B: $6 \times 3 = 18$.

So the chain sends $2 \rightarrow 18$.

55

A: $+3$ B: $\times 2$

Input 4 $\rightarrow$ A: $\rightarrow$ B:

58

A: $\div 3$ B: $+7$

Input 24 $\rightarrow$ A: $\rightarrow$ B:

56

A: $\times 4$ B: -5

Input 6 $\rightarrow$ A: $\rightarrow$ B:

59

A: $\times 2, +1$ B: $+4$

Input 5 $\rightarrow$ A: $\rightarrow$ B:

57

A: -2 B: $\times 5$

Input 9 $\rightarrow$ A: $\rightarrow$ B:

60

A: $+8$ B: $\div 4$

Input 12 $\rightarrow$ A: $\rightarrow$ B:

Lab 6B: Swap the Machines

Does A then B equal B then A? Usually not.

61

A: +5	B: $\times 2$	Starting input: 7	
A then B:	B then A:		Same?

62

A: $\times 3$	B: $\times 2$	Starting input: 4	
A then B:	B then A:		Same?

63

A: +6	B: -2	Starting input: 10	
A then B:	B then A:		Same?

64

A: $\times 4$	B: +3	Starting input: 5	
A then B:	B then A:		Same?

65. What do you suspect?

Which kinds of operations seemed safe to swap above? Which did not? This is a conjecture, not a rule you have to memorize.

Lab 7: Pattern Worlds

A sequence changes from one term to the next

Mission

A number machine maps an input to an output. A sequence is different: each term may tell you how to make the *next* term. Find the pattern and continue it.

66

4, 9, 14, 19, ,

Rule:

67

3, 6, 12, 24, ,

Rule:

68

50, 43, 36, 29, ,

Rule:

69

2, 5, 10, 13, 26, 29, ,

Rule:

70

80, 40, 44, 22, 26, 13

Describe the alternating rule in words:

71

1, 4, 9, 16, 25,

What do these numbers have in common?

Lab 7B: Pattern Traps

Do not assume every pattern is “add the same number”

72. Growing jumps

2, 4, 7, 11, 16, _____,

Hint: look at the amount added each time.

Explain:

73. Two patterns woven together

2, 20, 4, 18, 6, 16, 8, _____,

Explain what the odd-position terms do and what the even-position terms do.

74. Describe, don't just continue

1, 2, 4, 7, 11, 16, 22,...

Next two terms: _____,

Rule in words:

75. Make a deceptive sequence

Create a sequence with at least 7 terms that uses an alternating rule. Give it to someone without telling them the rule.

Sequence:

Secret rule:

Lab 8: Rule Matchmaker

Which rule belongs to which evidence?

Evidence	Rule bank
A. $2 \rightarrow 11$, $4 \rightarrow 19$, $6 \rightarrow 27$	1. $\times 2, +7$
B. $3 \rightarrow 13$, $5 \rightarrow 17$, $8 \rightarrow 23$	2. $\times 4, +3$
C. $2 \rightarrow 8$, $5 \rightarrow 14$, $9 \rightarrow 22$	3. $\times 3, -2$
D. $4 \rightarrow 12$, $7 \rightarrow 18$, $12 \rightarrow 28$	4. $\times 2, +4$
E. $3 \rightarrow 7$, $6 \rightarrow 16$, $10 \rightarrow 28$	5. $\times 3, +4$

76

Match each evidence set to a rule.

A $\rightarrow$ B $\rightarrow$ C $\rightarrow$ D $\rightarrow$ E $\rightarrow$

77. There is a problem in the rule bank

Look carefully: do all five evidence sets have exactly one matching rule in the bank? If not, explain what is wrong and suggest a repair.

Challenge Room 1

Mixed mysteries - no labels telling you what kind of puzzle it is

78

$2 \rightarrow 8$, $4 \rightarrow 14$, $7 \rightarrow 23$, $10 \rightarrow$

Rule:

79

Rule: $\times 5, -6$. A secret input produces 39. Find the input.

Input = Work:

80

Clues: $3 \rightarrow 8$, $5 \rightarrow 12$, $8 \rightarrow 18$, $10 \rightarrow 23$. Exactly one clue is broken.

Broken clue: Rule:

Repaired output:

81

Two suspects both fit $4 \rightarrow 14$.

A: $\times 3, +2$ B: $\times 2, +6$

Choose a useful test input and predict both outputs.

Input A: B:

82

A machine sends $1 \rightarrow 6$, $2 \rightarrow 11$, $3 \rightarrow 16$. A student says, "The rule is add 5." Explain the mistake and give a rule that fits all clues.

Challenge Room 2

Harder puzzles for detectives who want a fight

83. Three-step machine

Rule: $\times 2$, then $+3$, then $\times 4$.

Input 5 $\rightarrow$ output

Output 76 came from input . Show your backward steps:

84. Hidden chain

Machine A adds 2. Machine B is unknown. Together they send:

3 $\rightarrow$ 20, 5 $\rightarrow$ 28, 8 $\rightarrow$ 40.

What does Machine B do?

85. Input itself is a clue

A machine uses the rule “multiply the input by itself, then add 1.”

2 $\rightarrow$ 5, 3 $\rightarrow$ 10, 4 $\rightarrow$ 17.

What is 6 $\rightarrow$? What input gives output 26?

86. Compare two machines

P: $\times 3, +2$ Q: $\times 2, +8$

Find an input for which P and Q give the **same** output.

Input = Shared output =

87. Explain your method

Pick the hardest problem on this page. Explain how you attacked it, even if you made a wrong guess first.

Invent a Machine

Now you become the puzzle designer

Design rules

Good puzzle designers choose clues carefully. If your clues are too easy, the rule is obvious. If they are too vague, many rules fit. Your goal is to give just enough evidence to make the solver think.

88. Build a one-step machine

Secret rule:

Clue 1: →

Clue 2: →

Clue 3: →

Challenge input: → ?

89. Build a two-step machine

Secret rule:

Clue 1: →

Clue 2: →

Clue 3: →

Reverse challenge: ? →

90. Build a broken machine

Choose a real rule. Write four clues, but deliberately make exactly one output wrong. Give the puzzle to someone else and see whether they can repair it.

Challenge a Grown-Up

A page designed to start a mathematical argument

91. The tiny-evidence trap

Tell a grown-up: "My sequence starts 2, 4, 6. What comes next?"

If they say 8, ask: "How do you know?" Invent a different reasonable pattern whose fourth term is **not 8**.

92. Machine duel

Invent two different machines that both send $4 \rightarrow 20$ but disagree on input 7.

Machine A: $7 \rightarrow$

Machine B: $7 \rightarrow$

93. Make them reverse it

Invent a two-step machine and give the grown-up only the rule and an output. Make them find the input.

Rule:

Output:

Secret input:

Pattern Detective Field Notes

What did you discover about mathematical thinking?

The real victory

If you and your grown-up disagree about a pattern, do not ask “Who is right?” first. Ask:
What evidence would tell the rules apart?

94. A rule is convincing when...

95. When evidence is not enough, I can...

96. Working backward helps because...

97. My favorite puzzle in this workbook was number...

Puzzle:

Because:

98. Final invention

Create one puzzle that you think would fool the person who gave you this workbook.

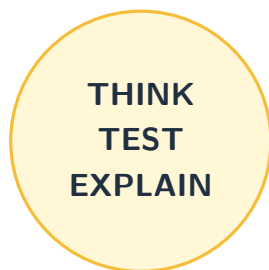
PATTERN DETECTIVE CERTIFICATE

This certifies that

has completed the Secret Machine Lab

and demonstrated the ability to

**notice patterns, propose rules, test evidence, work backward,
and challenge answers that are not fully proved.**



Date

Pattern Lab Mentor

Answer Key

For grown-ups, teachers, or detectives checking their own work

Labs 1 and 1B: 1–12

1. $+5$; 17. 2. $\times 3$; 33. 3. -7 ; 18.
4. $\div 2$; 15. 5. $\times 5$; 60. 6. $\div 4$; 15.
7. A. 8. B. 9. A. 10. A. 11. B. 12. A.
The explanations in 7–12 should check all clues, not just one.

Lab 2: 13–18

13. $\times 3, +1$; 25.
14. $\times 4, +1$; 41.
15. $\times 4, +1$; 41.
16. $\times 2, +2$; 26.
17. $\times 2, +3$; 63.
18. $\times 3, -2$; 58.

Notice something interesting

Problems 14 and 15 have the **same rule** even though their clue sets are different. Evidence does not need to look the same to point to the same underlying structure.

Answer Key

Order, reverse engineering, and missing information

19–22

19. A; input 9 gives 24.

20. Both! “+3, then $\times 3$ ” is $(x + 3) \times 3 = 3x + 9$, which is exactly the same result as “ $\times 3$, then +9.” This is a great discussion problem.

21. P: 18. Q: 10. Difference: 8.

22. Answers vary. Example: +4 then $\times 3$ versus $\times 3$ then +4 on input 2 gives 18 versus 10.

23–28

23. 7. 24. 8. 25. 8. 26. 13. 27. 24. 28. 36.

Strategy response: undo +8 first by subtracting 8, then undo $\times 6$ by dividing by 6.

29–38

29. 20. 30. 8. 31. 23. 32. 35.

33. 30. 34. 9. 35. 37. 36. 20.

37. $\times 3, +2$; $12 \rightarrow 38$.

38. Many answers. Example input 2 gives 11 under $\times 4, +3$, while “+6” would give 8, so $2 \rightarrow 11$ separates them.

Answer Key

Broken machines and evidence

39–44

- 39. Broken $6 \rightarrow 20$; rule $\times 3, +3$; it should be $6 \rightarrow 21$.
- 40. Broken $9 \rightarrow 35$; rule $\times 4, +2$; it should be $9 \rightarrow 38$.
- 41. Broken $10 \rightarrow 40$; rule $\times 4, +2$; it should be $10 \rightarrow 42$.
- 42. Broken $20 \rightarrow 45$; rule $\times 2, +2$; it should be $20 \rightarrow 42$.
- 43. No clue is actually broken: all fit $\div 2, +1$. This is an intentional meta-trap. A careful detective should challenge the directions rather than inventing an error.
- 44. Broken $15 \rightarrow 44$; rule $\times 3, -2$; it should be $15 \rightarrow 43$.

45–48

- 45. $6 \rightarrow 31$ is broken; repair to $6 \rightarrow 32$.
- 46. $12 \rightarrow 20$ is broken; repair to $12 \rightarrow 21$.
- 47. $30 \rightarrow 15$ is broken; repair to $30 \rightarrow 14$.
- 48. No. One clue cannot prove that rule. Many other rules send 2 to 9, such as $+7$ or $\times 3, +3$.

Answer Key

Not enough evidence and experimental design

49–54

49. Many answers. For $5 \rightarrow 20$: $\times 4$; $+15$; $\times 3$, $+5$ are three examples.
50. Almost any input other than 5 works. Example input 2: A predicts 8, B predicts 11.
51. Any input other than 3 works. Example input 4: P gives 13, Q gives 12.
52. Choose any input where the predictions differ. Input 2 is a bad choice because both rules give 7. Input 4 works: A gives 11, B gives 13. If the machine returns 11, that supports A and rules out B among these two suspects.
53. Both rules send 10 to 20, so the test cannot distinguish them. Example better input 4: A gives 14, B gives 8.
54. A good test input makes the competing rules predict **different outputs**. The farther apart or easier to check, the better.

Why 52 is sneaky

Two different rules can agree on more than one input. Students should discover that a test is useful only when the suspects make different predictions for that test.

Answer Key

Machine chains and swapping order

55–60

- 55. 7, then 14.
- 56. 24, then 19.
- 57. 7, then 35.
- 58. 8, then 15.
- 59. 11, then 15.
- 60. 20, then 5.

61–65

- 61. A then B: 24. B then A: 19. Not the same.
- 62. 24 and 24. Same.
- 63. 14 and 14. Same.
- 64. 23 and 32. Not the same.
- 65. Multiplication by whole-number factors can be swapped with multiplication; addition/subtraction by fixed amounts can also combine in either order. Mixing addition with multiplication usually changes the result. This is an observation students can explore, not a formal proof requirement.

Answer Key

Sequences and pattern traps

66–71

- 66. 24, 29; add 5.
- 67. 48, 96; multiply by 2.
- 68. 22, 15; subtract 7.
- 69. 58, 61; alternate $+3$, then $\times 2$.
- 70. Alternate $\div 2$, then $+4$.
- 71. 36; the terms are square numbers: $1^2, 2^2, 3^2, 4^2, 5^2, 6^2$.

72–75

- 72. 22, 29. Add 2, then 3, then 4, then 5, then 6, then 7.
- 73. 14, 10. Odd-position terms go 2,4,6,8,10 (add 2). Even-position terms go 20,18,16,14 (subtract 2).
- 74. 29, 37. The additions grow: $+1, +2, +3, +4, +5, +6, +7, +8, \dots$
- 75. Answers vary.

Answer Key

Rule matchmaker and mixed challenges

76–77

A matches 2 ($\times 4, +3$).

B matches 1 ($\times 2, +7$).

C matches 4 ($\times 2, +4$).

D also matches 4 ($\times 2, +4$).

E matches 3 ($\times 3, -2$).

So the bank is flawed: C and D share the same rule and rule 5 ($\times 3, +4$) is unused. A repair could change D's evidence to $2 \rightarrow 10$, $4 \rightarrow 16$, $6 \rightarrow 22$, which matches rule 5.

78–82

78. Rule $\times 3, +2$; output 32.

79. Input 9, because $39 + 6 = 45$ and $45 \div 5 = 9$.

80. Intended rule is $\times 2, +2$. The broken clue is $10 \rightarrow 23$; repair it to $10 \rightarrow 22$.

81. Any input other than 4 works. Example input 2: A gives 8, B gives 10.

82. "Add 5" fits only the first step from input 1 to output 6, but it fails the other pairs. A fitting rule is $\times 5, +1$.

Answer Key

Challenge Room 2 and invention pages

83–87

83. Input 5: $5 \times 2 = 10$, $+3 = 13$, $\times 4 = 52$. For output 76: $76 \div 4 = 19$, $19 - 3 = 16$, $16 \div 2 = 8$. Input 8.

84. After Machine A, the inputs to B are 5, 7, 10 and outputs are 20, 28, 40. So B is $\times 4$.

85. $6 \rightarrow 37$. Output 26 comes from input 5 because $5 \times 5 + 1 = 26$.

86. Solve by testing or reasoning: input 6 works. P gives 20 and Q gives 20.

87. Explanation varies; reward a clear strategy, including useful failed guesses.

88–98

These are design and reflection problems, so answers vary. Check that invented clues actually follow the stated secret rule and that “broken machine” puzzles have exactly the intended inconsistency. For 91, many non-8 fourth terms are defensible if the student can state a coherent rule. Example: 2, 4, 6, 12, 14, 28, ... can follow “+2, +2, $\times 2$ ” repeating, making the fourth term 12.

What to praise

The strongest answers are not necessarily the fastest. Praise students who test a rule against every clue, notice bad directions, ask for more evidence, or invent a test that distinguishes competing explanations.

For the Grown-Up

What this workbook is secretly teaching

Under the hood

Although the pages look like number puzzles, the deeper skills are foundational mathematical habits:

- **Function thinking:** an input is transformed into an output by a consistent rule.
- **Generalization:** the student describes a process that works beyond the examples shown.
- **Inverse reasoning:** undoing operations in reverse order.
- **Composition:** understanding that one transformation can feed another.
- **Non-commutativity:** discovering that changing operation order can change results.
- **Hypothesis testing:** proposing a rule and checking it against evidence.
- **Underdetermination:** recognizing that limited evidence may support multiple explanations.
- **Experimental design:** choosing a new input that distinguishes competing rules.
- **Mathematical communication:** explaining why a rule fits, not merely announcing an answer.

A useful way to respond when a child is stuck

Instead of saying “try again,” ask one of these:

“What changed from input to output?” “Does your idea fit the second clue?”

“Can you undo the last step?” “What input would help you tell those two ideas apart?”

“Can you invent a different rule that fits the same clue?”

The goal is not to guess the author’s mind.

The goal is to reason from evidence.