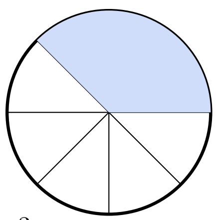
Fractions: Meaning Before Machinery

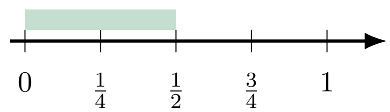
A visual, verbal, concept-first workbook

Addition Subtraction Multiplication Division

Equivalent Fractions Ratios Mixed Numbers



$\frac{3}{8}$ of a pizza



$\frac{1}{2}$ is a distance

$\frac{5}{8}$ is 5 pieces when the whole is cut into 8 equal pieces

The goal is not to memorize mysterious rules.

The goal is to look at a fraction problem and understand what it is *saying.*

Name: ______________________________

Designed for a strong verbal/conceptual learner who needs computation to become meaningful.

How to use this workbook

Guide note

This workbook is intentionally **not** a giant stack of near-identical problems. The learner should spend more time **explaining, drawing, predicting, and translating** than racing through arithmetic. A child can have excellent mathematical ideas and still freeze when notation has not yet become meaningful. The cure is to connect the symbols to language, pictures, and quantities until the notation becomes readable.

Big idea

A fraction problem is a sentence written in mathematical shorthand. Before calculating, translate it into ordinary language.

The five-question routine

Whenever you see a problem, ask these in order:

1. **What is the whole?** A pizza? One mile? One cup? One rectangle? The number 1?
2. **What does the denominator mean?** It tells the size/name of the pieces: thirds, fifths, eighths, and so on.
3. **What does the numerator mean?** It tells how many of those pieces we have.
4. **What is the operation saying?** Add = combine. Subtract = remove/compare. Multiply = take "of" or scale. Divide = share or ask "how many groups fit?"
5. **What size answer should I expect?** Bigger? Smaller? About 1? About 10? This catches many mistakes before they happen.

Say it in words

Do not begin with "What rule do I use?" Begin with **"What does this mean?"**

A 15-minute session

3 minutes	Read one explanation and describe the picture aloud.
5 minutes	Do 2-4 problems, but explain each one before calculating.
4 minutes	Draw one problem in a different way: strip, number line, groups, or area.
3 minutes	Do one "cold" problem from the mixed practice section.

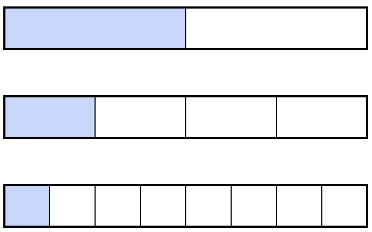
Part I: What a fraction actually is

Big idea

A fraction is not just “one number on top of another.” It can describe a **part of a whole**, a **point on a number line**, a **division**, or a **scaling instruction**.

1. The denominator names the size of the pieces

Look at the same-size whole cut in different ways.



$\frac{1}{2}$: one piece when the whole is cut into 2 equal pieces

$\frac{1}{4}$: one piece when the whole is cut into 4 equal pieces

$\frac{1}{8}$: one piece when the whole is cut into 8 equal pieces

Think before you calculate

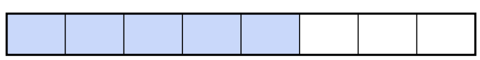
Which is larger: $\frac{1}{4}$ or $\frac{1}{8}$? Do not calculate. Explain using the meaning of the denominator.

____

Say it in words

A larger denominator means the same whole has been chopped into **more pieces**, so each individual piece is **smaller**. This is why $\frac{1}{8} < \frac{1}{4}$.

2. The numerator counts how many pieces you have



The denominator 8 says the whole has been divided into eighths. The numerator 5 says we have five of those eighths.

Your turn

For each fraction, write a sentence beginning, “If the whole is cut into ...”

1. $\frac{3}{7}$

____

2. $\frac{11}{12}$

____

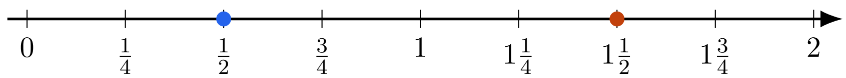
3. $\frac{2}{5}$

____

Fractions live on the number line

Big idea

A fraction is a **number**. It has a location and a size. Thinking of fractions as points on a line prevents many common mistakes.



$\frac{1}{2}$ is halfway from 0 to 1. $1\frac{1}{2}$ is halfway from 1 to 2. Mixed numbers are not exotic: they are ordinary numbers larger than 1.

Think before you calculate

Without calculating decimals, place each roughly on a number line from 0 to 2: $\frac{1}{8}$, $\frac{7}{8}$, $\frac{5}{4}$, $1\frac{7}{8}$.

Fractions are also division

The fraction bar itself means division:

$$\frac{3}{4} = 3 \div 4.$$

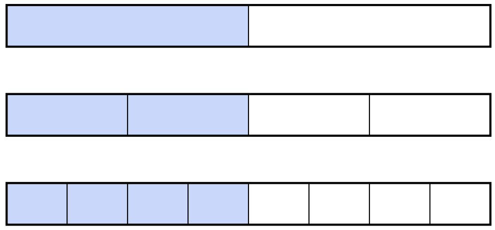
So $\frac{3}{4}$ can mean, “Take 3 things and share them equally among 4 groups.” Each group gets $\frac{3}{4}$ of one thing.

Your turn

Translate each fraction into division words.

1. $\frac{2}{5}$ means ______________ divided by ______________.
2. $\frac{7}{6}$ means ______________ divided by ______________.
3. $\frac{13}{4}$ means ______________ divided by ______________.

Equivalent fractions: same amount, different name



Big idea

$\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$ are not three different amounts. They are **three names for the same amount**.

Why does multiplying numerator and denominator by the same number preserve the value?

$$\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}.$$

But $\frac{2}{2} = 1$, so we have really multiplied by 1. We changed the **name**, not the **size**.

The word machine

Think of equivalent fractions as changing units. Saying "half a foot" and "six inches" changes the unit name, but not the physical length.

Your turn

Draw or reason your way to three equivalent names for each fraction.

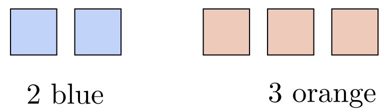
1. $\frac{1}{3}$ = __________ = __________ = __________
2. $\frac{3}{4}$ = __________ = __________ = __________
3. $\frac{2}{5}$ = __________ = __________ = __________

Fractions can describe ratios: “for every”

Big idea

A **ratio** compares two quantities. The ratio $2 : 3$ is read “2 to 3” and can be said in words as **“2 for every 3.”** The fraction $\frac{2}{3}$ can express that same first-to-second comparison as a quotient.

Suppose a tray has 2 blue tiles and 3 orange tiles:



We can compare blue with orange:

$$2 : 3 \qquad \text{or} \qquad \frac{2}{3}.$$

Both say: **for every 2 blue tiles, there are 3 orange tiles.**

The word machine

A ratio always has a direction. $2 : 3$ means *first quantity to second quantity.* Reversing the order gives $3 : 2$, which answers a different comparison.

Say it in words

When you see $a : b$, ask: **“a of what, for every b of what?”** Naming the quantities keeps ratio notation from becoming meaningless symbols.

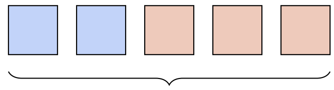
Your turn

Say each ratio aloud using “for every.” Then write it as a fraction comparison.

1. 4 red markers for every 7 black markers: $4 : 7 =$ __________
2. 3 dogs for every 5 cats: $3 : 5 =$ __________
3. 12 wins for every 4 losses: $12 : 4 =$ __________

Part-to-part and part-to-whole are different questions

Use the same 2 blue tiles and 3 orange tiles. There are **5 tiles total**.



5 tiles altogether

Question	Meaning	Notation
Blue : orange	Compare one *part* with another *part*.	2 : 3
Blue : total	Compare the blue part with the *whole collection*.	2 : 5
Fraction blue	What fraction of all 5 tiles are blue?	$\frac{2}{5}$

Common trap

If there are 2 blue and 3 orange tiles, the fraction that are blue is **not** $\frac{2}{3}$. The denominator for "fraction of the total" must count **all 5 tiles**, so the fraction is $\frac{2}{5}$.

Big idea

The numbers do not tell you the relationship by themselves. The **words tell you what is being compared**. "Blue to orange" and "blue out of all tiles" are different questions.

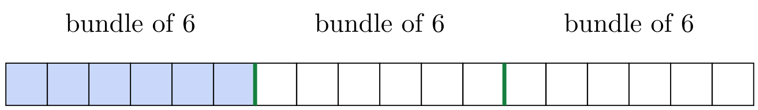
Your turn

A basket contains 4 green apples and 6 red apples. There are 10 apples total.

1. Green : red = ____________
2. Red : green = ____________
3. Green : total = ____________
4. Fraction of all apples that are green = ____________

Reducing fractions: same relationship, bigger bundles

Consider $\frac{6}{18}$. A rule might say "divide top and bottom by 6." But what does that *mean*?



$\frac{6}{18}$ means 6 small pieces out of 18

Now stop counting individual pieces. Count **bundles of 6** instead:

$$6 = 1 \times 6, \qquad 18 = 3 \times 6.$$

So the same relationship becomes

$$\frac{6}{18} = \frac{1 \text{ bundle}}{3 \text{ bundles}} = \frac{1}{3}.$$

Big idea

Reducing does not change the amount or relationship. It changes the size of the unit you are counting. Instead of counting tiny pieces one by one, you count equal bundles.

This is why we divide numerator and denominator by the **same** number:

$$\frac{6}{18} = \frac{6 \div 6}{18 \div 6} = \frac{1}{3}.$$

We have regrouped both quantities using the same bundle size.

Say it in words

Read $\frac{8}{12} = \frac{2}{3}$ as: "Eight out of twelve has the same relationship as two out of three, because both numbers can be regrouped into bundles of four."

Your turn

For each fraction, name a common bundle size and reduce.

1. $\frac{8}{12}$: bundle size _______; reduced fraction __________
2. $\frac{15}{25}$: bundle size _______; reduced fraction __________
3. $\frac{14}{21}$: bundle size _______; reduced fraction __________

Common factors, GCF, and lowest terms

A **factor** is a whole number that divides another whole number evenly. A **common factor** is a bundle size that works for *both* numerator and denominator.

For $\frac{12}{18}$:

Factors of 12:	$1, 2, 3, 4, 6, 12$
Factors of 18:	$1, 2, 3, 6, 9, 18$
Common factors:	$1, 2, 3, 6$
Greatest common factor (GCF):	6

Using the largest shared bundle size reduces in one step:

$$\frac{12}{18} = \frac{12 \div 6}{18 \div 6} = \frac{2}{3}.$$

Big idea

A fraction is in **lowest terms** when numerator and denominator have no common factor larger than 1. In bundle language: **there is no larger whole-number bundle size that fits evenly into both counts.**

You do not always need the GCF immediately. Repeated reduction also works:

$$\frac{24}{36} \xrightarrow{\div 2} \frac{12}{18} \xrightarrow{\div 6} \frac{2}{3}.$$

Or use the GCF 12 at once:

$$\frac{24}{36} \xrightarrow{\div 12} \frac{2}{3}.$$

Same destination, different route.

Think before you calculate

How can you tell that $\frac{2}{3}$ is finished? Explain without saying “because I know it is.”

____

Your turn

Find a useful common factor (the GCF if you can) and reduce to lowest terms.

1. $\frac{18}{24} =$ ____________
2. $\frac{21}{49} =$ ____________
3. $\frac{36}{60} =$ ____________
4. $\frac{45}{100} =$ ____________

One relationship, three notations: fraction, division, ratio

The same two quantities can be written in related ways, depending on what you want to emphasize.

Form	Notation	What it emphasizes
Fraction	$\frac{300}{9000}$	A quotient or first quantity compared with the second.
Division	$300 \div 9000$	The action of dividing 300 by 9000.
Ratio	$300 : 9000$	"300 to 9000" or "300 for every 9000."

Now ask: **What common bundle size fits both quantities?** Here, 300 itself works.

$$300 = 1 \times 300 \qquad \text{and} \qquad 9000 = 30 \times 300$$

So measured in **300-sized bundles**, the two counts are **1 and 30**.

Therefore:

$$\frac{300}{9000} = \frac{1}{30}, \qquad 300 : 9000 = 1 : 30.$$

Common trap

Be precise with the notation. The **fraction** reduces to $\frac{1}{30}$. The **ratio** reduces to $1 : 30$. They express the same first-to-second relationship, but the symbols are not interchangeable punctuation.

Say it in words

"There is 1 unit of the first quantity for every 30 units of the second." That sentence is the meaning hiding inside $1 : 30$.

Your turn

Reduce each comparison. Write both the reduced fraction and the reduced ratio.

1. 20 to 100: $\frac{20}{100} =$ __________ and $20 : 100 =$ ____________
2. 45 to 60: $\frac{45}{60} =$ __________ and $45 : 60 =$ ____________
3. 300 to 9000: $\frac{300}{9000} =$ __________ and $300 : 9000 =$ ____________

Mixed numbers and improper fractions

Big idea

A mixed number such as $2\frac{3}{4}$ means **two whole units plus three fourths of another unit**.

Two whole bars and three fourths

Count the fourths: each whole contains 4 fourths. Two wholes contain 8 fourths, plus 3 more makes 11 fourths.

$$2\frac{3}{4} = \frac{11}{4}.$$

Say it in words

To convert a mixed number to an improper fraction, do not memorize "multiply-add" as a magic chant. Think: **How many denominator-sized pieces are hiding in the wholes?**

Example: $3\frac{1}{5}$ means 3 wholes, and each whole has 5 fifths. That is 15 fifths, plus 1 fifth = 16 fifths.

Your turn

Convert by writing a sentence first.

1. $1\frac{2}{3} = \frac{\square}{3}$. Sentence: ______________________________
2. $4\frac{3}{8} = \frac{\square}{8}$. Sentence: ______________________________
3. $7\frac{3}{5} = \frac{\square}{5}$. Sentence: ______________________________

Part II: Addition and subtraction

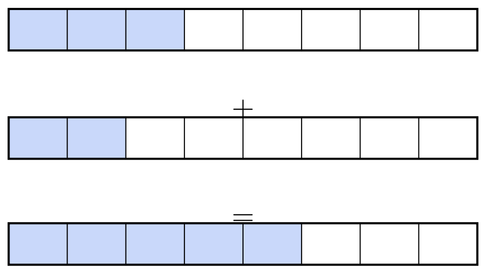
Big idea

Addition and subtraction require **same-size pieces**. You can combine 3 quarters with 2 quarters. You cannot directly combine “3 quarters” with “2 fifths” because the pieces are different sizes.

Same denominator: the easy case

$$\frac{3}{8}+\frac{2}{8}=\frac{5}{8}.$$

Read it aloud: “Three eighths plus two eighths equals five eighths.” The object being counted is **eighths**, just as “3 apples + 2 apples = 5 apples.”



Common trap

Do **not** add denominators. $\frac{3}{8}+\frac{2}{8}$ is not $\frac{5}{16}$. If you start with eighth-size pieces, combining them does not magically turn them into sixteenth-size pieces.

Your turn

Before calculating, circle the word that names the pieces.

1. $\frac{2}{9}+\frac{4}{9}=$
2. $\frac{7}{10}-\frac{3}{10}=$
3. $\frac{5}{12}+\frac{1}{12}=$
4. $\frac{11}{15}-\frac{4}{15}=$

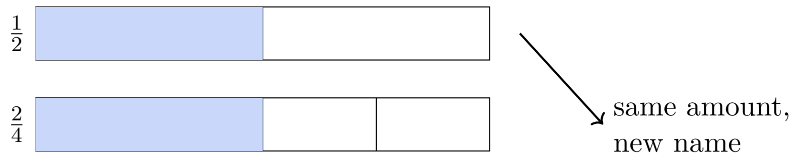
Why common denominators exist

Imagine this sentence:

"Add 1 half-liter to 1 quarter-liter."

You can do it, but it is easier if both quantities are described in the same unit. One half-liter is two quarter-liters. So:

$$\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}.$$

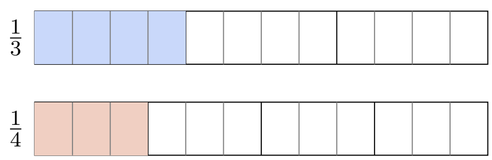


Say it in words

A **common denominator** is a common unit. It makes both fractions speak the same language.

A visual method for $\frac{1}{3} + \frac{1}{4}$

The thirds and fourths do not line up. Cut both wholes into twelfths. Why 12? Because 12 can be divided evenly into groups of 3 and groups of 4.



$\frac{1}{3} = \frac{4}{12}$ and $\frac{1}{4} = \frac{3}{12}$, so together they make $\frac{7}{12}$.

Finding a common denominator without guessing

There are two useful strategies.

Strategy A: list multiples

For denominators 6 and 8:

$$6, 12, 18, 24, 30, \ldots$$

$$8, 16, 24, 32, \ldots$$

The first shared multiple is 24, so 24 is the **least common denominator**.

Strategy B: use a denominator you already see

If one denominator is already a multiple of the other, use the larger one.

$$\frac{3}{8} + \frac{1}{4}.$$

Since 8 is a multiple of 4, rename $\frac{1}{4}$ as eighths: $\frac{1}{4} = \frac{2}{8}$. Then add.

Your turn

Write a useful common denominator. You do not need to finish the addition yet.

1. $\frac{1}{2} + \frac{1}{5}$: ______
2. $\frac{3}{4} + \frac{1}{6}$: ______
3. $\frac{5}{8} + \frac{3}{10}$: ______
4. $\frac{7}{12} - \frac{1}{6}$: ______
5. $\frac{2}{9} + \frac{5}{6}$: ______
6. $\frac{3}{14} + \frac{2}{7}$: ______

Think before you calculate

Explain why any common multiple works, even if it is not the least common multiple. Why might the least one still be convenient?

____

____

Addition with unlike denominators: translate, rename, combine

Example:

$$\frac{2}{5} + \frac{3}{8}.$$

1. **Translate:** two fifths plus three eighths.
2. **Notice the problem:** fifths and eighths are different-size pieces.
3. **Choose a common unit:** fortieths.
4. **Rename:** $\frac{2}{5} = \frac{16}{40}$ and $\frac{3}{8} = \frac{15}{40}$.
5. **Combine:** $\frac{16}{40} + \frac{15}{40} = \frac{31}{40}$.

The word machine

This is not a trick. You are doing the same thing as converting 2 feet and 9 inches into a common unit before combining lengths.

Your turn

For each problem, do all four moves: **say it, predict the size, rename, calculate**.

1. $\frac{1}{2} + \frac{1}{3}$ Prediction: ____________
   Work: ____
2. $\frac{3}{4} + \frac{2}{5}$ Prediction: ____________
   Work: ____
3. $\frac{7}{8} - \frac{1}{3}$ Prediction: ____________
   Work: ____
4. $\frac{5}{6} - \frac{1}{4}$ Prediction: ____________
   Work: ____

Adding and subtracting mixed numbers

Example:

$$2\frac{3}{4} + 1\frac{2}{3}.$$

There are two sensible approaches.

Approach 1: wholes with wholes, fractions with fractions

$$(2+1) + \left(\frac{3}{4} + \frac{2}{3}\right).$$

Rename the fractional pieces in twelfths:

$$3 + \left(\frac{9}{12} + \frac{8}{12}\right) = 3 + \frac{17}{12} = 4\frac{5}{12}.$$

Approach 2: convert everything to improper fractions

$$2\frac{3}{4} = \frac{11}{4}, \qquad 1\frac{2}{3} = \frac{5}{3}.$$

Then use a common denominator and add.

Big idea

The two approaches are mathematically the same. Choose the representation that makes the meaning easiest to see.

A problem from the screenshots: $12 + 2\frac{3}{4}$

This asks: "Start at 12 and move forward another 2 and three quarters."

$$12 + 2\frac{3}{4} = 14\frac{3}{4}.$$

No special fraction machinery is needed because the whole number 12 simply combines with the 2 wholes.

Your turn

Explain each in words before computing.

1. $5 + 3\frac{1}{8} =$ ________________
   Meaning: ____

2. $7\frac{1}{2} + 2\frac{1}{4} =$ ________________
   Meaning: ____

3. $9\frac{1}{3} - 4\frac{5}{6} =$ ________________
   Prediction: ____

Mixed-number subtraction: what "borrowing" really means

Suppose you see:

$$5\frac{1}{4} - 2\frac{2}{3}.$$

The fractional part $\frac{1}{4}$ is smaller than $\frac{2}{3}$, so you cannot remove two thirds from one quarter. But you have five whole units. You can **trade one whole for fraction pieces**.

Big idea

"Borrowing" is really **renaming one whole**. If your denominator is 4, one whole is $\frac{4}{4}$. If your denominator is 12, one whole is $\frac{12}{12}$.

First put the fractions into a common unit:

$$\frac{1}{4} = \frac{3}{12}, \qquad \frac{2}{3} = \frac{8}{12}.$$

Now rename one of the 5 wholes as $\frac{12}{12}$:

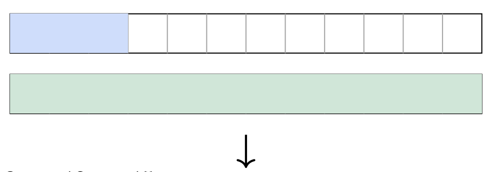
$$5\frac{3}{12} = 4\frac{15}{12}.$$

Now the subtraction is readable:

$$4\frac{15}{12} - 2\frac{8}{12} = 2\frac{7}{12}.$$

before trade: $\frac{3}{12}$

one whole: $\frac{12}{12}$



$\frac{3}{12} + \frac{12}{12} = \frac{15}{12}$ available to subtract from

Say it in words

You did not create extra amount. You changed the way the same amount was named so the subtraction could happen.

Your turn

For each problem, decide whether a trade is needed. If it is, write the traded form before subtracting.

1. $3\frac{1}{5} - 1\frac{3}{4} =$ ________________

2. $6\frac{2}{3} - 2\frac{5}{6} =$ ________________

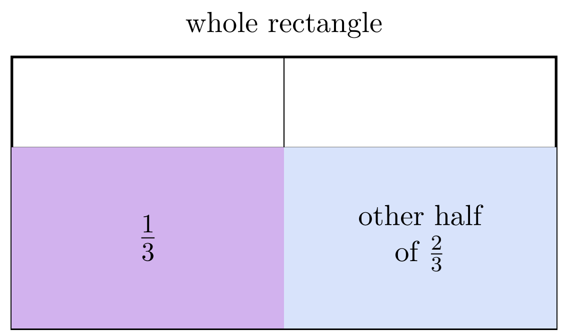
3. $8\frac{1}{4} - 3\frac{7}{8} =$ ________________

Part III: Multiplication means "of" and scaling

Big idea

When fractions multiply, the word **of** is often the best translation. $\frac{1}{2} \times \frac{2}{3}$ means "one half **of** two thirds."

Area picture: $\frac{1}{2} \times \frac{2}{3}$



The rectangle has 6 equal small regions. The overlap contains 2 of them, so:

$$\frac{1}{2} \times \frac{2}{3} = \frac{2}{6} = \frac{1}{3}.$$

Say it in words

Numerator times numerator counts the overlapping pieces. Denominator times denominator tells how finely the whole has been partitioned.

Your turn

Predict first: should the answer be smaller than both fractions, between them, or larger?

1. $\frac{1}{2} \times \frac{3}{4}$ prediction: ______________________ result: ____________
2. $\frac{2}{3} \times \frac{3}{5}$ prediction: ______________________ result: ____________
3. $\frac{4}{5} \times \frac{7}{8}$ prediction: ______________________ result: ____________

Why multiplying by a proper fraction makes things smaller

Suppose you have 12 apples.

$$\frac{1}{2} \times 12 = 6.$$

You are taking only half of the original quantity. Likewise:

$$\frac{3}{4} \times 12 = 9,$$

which is still less than 12.

Think before you calculate

If you multiply a positive number by $\frac{1}{5}$, should the answer be bigger or smaller? Why? What if you multiply by $1\frac{1}{5}$?

____

Big idea

Multiplication is not always "make bigger." Multiplication is **scaling**. A scale factor below 1 shrinks; a scale factor above 1 enlarges.

Screenshot problem: $\frac{1}{3} \times \frac{1}{3}$

Read it as: "one third of one third." If a whole is divided into thirds, then one of those thirds is divided into thirds again. The result is one ninth of the original whole.

$$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}.$$

Screenshot problem: $\frac{2}{5} \times \frac{5}{8}$

$$\frac{2}{5} \times \frac{5}{8} = \frac{10}{40} = \frac{1}{4}.$$

Notice the cancellation hiding in the picture: the 5 in "fifths" and the 5 in "five eighths" partly undo each other.

Your turn

Compute and then explain the result using the phrase "a fraction of another fraction."

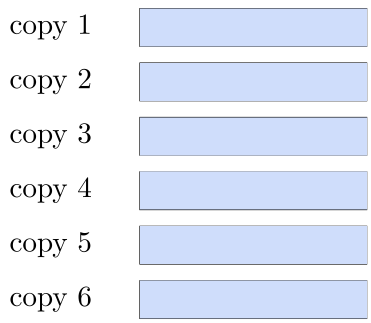
1. $\frac{2}{3} \times \frac{3}{4}$
2. $\frac{4}{5} \times \frac{1}{2}$
3. $\frac{3}{7} \times \frac{14}{15}$
4. $\frac{5}{8} \times \frac{2}{5}$

Whole number times a mixed number

Screenshot-style example:

$$6 \times 2\frac{1}{3}.$$

This means six copies of $2\frac{1}{3}$.



Verbally: six copies of 2 wholes gives 12 wholes. Six copies of $\frac{1}{3}$ gives $\frac{6}{3} = 2$ more wholes. Total: 14.

$$6 \times 2\frac{1}{3} = 14.$$

You can also convert $2\frac{1}{3} = \frac{7}{3}$ and multiply:

$$6 \times \frac{7}{3} = \frac{42}{3} = 14.$$

Say it in words

The improper-fraction method is compact, but the repeated-addition interpretation explains **why** it works.

Your turn

Use two methods on one problem of your choice.

1. $4 \times 1\frac{1}{2} =$ ______________

2. $3 \times 2\frac{3}{4} =$ ______________

3. $5 \times 1\frac{2}{5} =$ ______________

Method 1 explanation:

____

Method 2 explanation:

____

Simplifying before multiplying

Consider:

$$\frac{2}{5} \times \frac{5}{8}.$$

If we multiply first, we get $\frac{10}{40}$ and simplify to $\frac{1}{4}$.

But because multiplication lets us regroup factors, we can simplify first:

$$\frac{2 \cdot 5}{5 \cdot 8}.$$

The factor 5 appears on top and bottom, so it cancels because $\frac{5}{5} = 1$:

$$\frac{2 \cdot \cancel{5}}{\cancel{5} \cdot 8} = \frac{2}{8} = \frac{1}{4}.$$

Big idea

Canceling is not "crossing out numbers because a rule says so." You are removing a factor of 1.

Your turn

Circle factors that can cancel before multiplying.

1. $\frac{3}{7} \times \frac{14}{15}$
2. $\frac{5}{8} \times \frac{12}{25}$
3. $\frac{9}{10} \times \frac{5}{12}$
4. $\frac{11}{18} \times \frac{6}{22}$

Part IV: Division has two meanings

Big idea

Division answers one of two questions:

1. **Sharing:** If I split this amount into n equal groups, how much is in each group?
2. **Measurement:** How many groups of this size fit into that amount?

Whole-number examples

$12 \div 3 = 4$ can mean:

- Share 12 cookies among 3 people: 4 cookies each.
- How many groups of 3 cookies fit into 12 cookies? 4 groups.

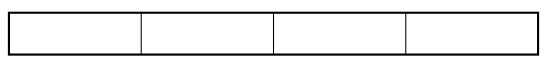
With fractions, the **measurement** meaning is especially powerful.

Screenshot problem: $1 \div \frac{1}{4}$

Read it as:

"How many quarter-size pieces fit inside 1 whole?"

Look at the strip:



There are 4 quarter-size pieces, so:

$$1 \div \frac{1}{4} = 4.$$

Say it in words

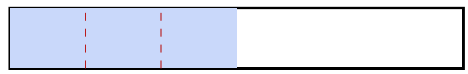
Dividing by a fraction can make the answer **bigger** because you are asking how many small pieces fit inside the starting amount.

Fraction divided by a whole number

Screenshot problem:

$$\frac{1}{2} \div 3.$$

Use the **sharing** meaning: "Share one half equally among 3 groups."



the original $\frac{1}{2}$ split into 3 equal shares

The whole is now effectively divided into 6 equal pieces, and each group gets 1 piece:

$$\frac{1}{2} \div 3 = \frac{1}{6}.$$

Big idea

Dividing by a whole number usually makes a positive quantity smaller because you are sharing it into more groups.

Your turn

Draw a strip and solve.

1. $\frac{3}{4} \div 2 =$ ______________

2. $\frac{2}{5} \div 3 =$ ______________

3. $\frac{7}{8} \div 7 =$ ______________

Fraction divided by fraction: "How many fit?"

Screenshot problem:

$$\frac{4}{5} \div \frac{3}{10}.$$

Translate it:

$$\text{“How many groups of } \tfrac{3}{10} \text{ fit inside } \tfrac{4}{5}\text{?”}$$

First make the pieces speak the same language:

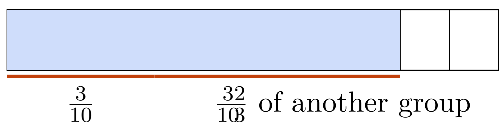
$$\frac{4}{5} = \frac{8}{10}.$$

Now ask: how many groups of 3 tenths fit in 8 tenths?

$$8 \div 3 = 2\frac{2}{3}.$$

So:

$$\frac{4}{5} \div \frac{3}{10} = 2\frac{2}{3}.$$



Say it in words

This picture explains the numerical answer $2\frac{2}{3}$: two full groups fit, plus two thirds of a third group.

Where “keep-change-flip” comes from

The shortcut is useful only after its meaning is understood.

Take:

$$\frac{4}{5} \div \frac{3}{10}.$$

Division by $\frac{3}{10}$ asks how many $\frac{3}{10}$-sized units fit. A unit of size $\frac{3}{10}$ fits $\frac{10}{3}$ times into one whole. Therefore it fits $\frac{4}{5}$ of that many times into $\frac{4}{5}$ of a whole:

$$\frac{4}{5} \times \frac{10}{3} = \frac{40}{15} = \frac{8}{3} = 2\frac{2}{3}.$$

Big idea

The reciprocal $\frac{10}{3}$ is the number of $\frac{3}{10}$-sized units per whole. Multiplying by the reciprocal converts “amount of whole” into “number of divisor-sized groups.”

So the algorithm

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

is not magic. It is a **unit conversion**.

Common trap

Do not flip the first fraction. The reciprocal belongs to the **size of the groups you are counting** - the divisor.

Your turn

For each, write the measurement question before calculating.

1. $\frac{3}{4} \div \frac{1}{8}$ means “How many ________ fit in ________?” Answer: __________
2. $\frac{2}{3} \div \frac{4}{9}$ means “How many ________ fit in ________?” Answer: __________
3. $\frac{5}{6} \div \frac{5}{12}$ means “How many ________ fit in ________?” Answer: __________

The "same fraction divided by itself" idea

Screenshot problem:

$$\frac{4}{5} \div \frac{4}{5}.$$

Ask: "How many groups of size $\frac{4}{5}$ fit inside an amount of $\frac{4}{5}$?" Exactly one.

$$\frac{4}{5} \div \frac{4}{5} = 1.$$

This is the fraction version of $7 \div 7 = 1$ or $100 \div 100 = 1$.

Big idea

Any nonzero number divided by itself equals 1 because the entire amount contains exactly one copy of itself.

Your turn

Without doing arithmetic, answer and explain.

1. $\frac{7}{13} \div \frac{7}{13} =$ __________.
   Why? ____

2. $5\frac{2}{3} \div 5\frac{2}{3} =$ __________.
   Why? ____

3. $0.125 \div 0.125 =$ __________.
   Why? ____

Mixed-number division

Screenshot-style example:

$$7\frac{3}{5} \div 3\frac{1}{3}.$$

Do not attack the symbols first. Estimate.

$7\frac{3}{5}$ is about 7.6. $3\frac{1}{3}$ is about 3.3. How many 3.3-sized groups fit in 7.6? A little more than 2. So an answer near 2 is plausible; answers like 21 or 24 are impossible.

Now convert to pieces that are easy to divide:

$$7\frac{3}{5} = \frac{38}{5}, \qquad 3\frac{1}{3} = \frac{10}{3}.$$

Then:

$$\frac{38}{5} \div \frac{10}{3} = \frac{38}{5} \times \frac{3}{10} = \frac{114}{50} = \frac{57}{25} = 2\frac{7}{25}.$$

Say it in words

The estimate came **before** the algorithm. That is how you know the final answer makes sense.

Your turn

Estimate first, then solve.

1. $5\frac{1}{2} \div 2\frac{1}{4}$ estimate: ____________ exact: ____________
2. $9\frac{3}{4} \div 1\frac{1}{2}$ estimate: ____________ exact: ____________
3. $4\frac{2}{5} \div 2\frac{1}{5}$ estimate: ____________ exact: ____________

Part V: Read the problem before doing the problem

Big idea

This section uses problems like the ones that felt opaque. The first task is not to calculate. The first task is to identify what the symbols are asking.

Problem: $\frac{1}{2} + \frac{1}{2}$

Say it in words

"One half plus another one half." Two halves make one whole.

$$\frac{1}{2} + \frac{1}{2} = 1.$$

Problem: $\frac{1}{2} \div 3$

Say it in words

"Share one half equally among three groups." Each group gets one sixth.

$$\frac{1}{2} \div 3 = \frac{1}{6}.$$

Problem: $1 \div \frac{1}{4}$

Say it in words

"How many quarter-size pieces fit in one whole?" Four.

$$1 \div \frac{1}{4} = 4.$$

Problem: $\frac{4}{5} \div \frac{4}{5}$

Say it in words

"How many copies of four fifths fit inside four fifths?" One.

$$\frac{4}{5} \div \frac{4}{5} = 1.$$

More problem-reading practice

Problem: $\frac{1}{2} \times \frac{2}{3}$

Say it in words

"One half of two thirds." Result: one third.

Problem: $\frac{2}{5} \times \frac{5}{8}$

Say it in words

"Two fifths of five eighths." Result: one fourth.

Problem: $\frac{1}{3} \times \frac{1}{3}$

Say it in words

"One third of one third." Result: one ninth.

Problem: $6 \times 2\frac{1}{3}$

Say it in words

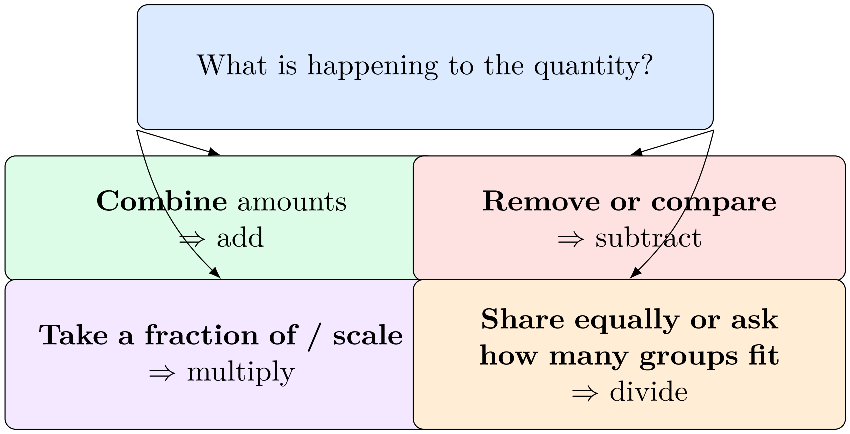
"Six copies of two and one third." Six copies of 2 is 12; six copies of one third is 2; total 14.

Problem: $\frac{4}{5} \div \frac{3}{10}$

Say it in words

"How many three-tenths-sized groups fit into four fifths?" Since four fifths is eight tenths, we ask how many 3s fit into 8: $2\frac{2}{3}$.

The operation detector



Language clues

Operation	Common phrases	Deep meaning
Add	total, altogether, combined, more	put quantities together
Subtract	left, difference, how much more, remaining	remove or compare
Multiply	of, times as much, scale, each of several groups	repeated groups or scaling
Divide	shared equally, per, how many groups, how many fit	sharing or measuring groups

Your turn

Do **not** calculate. Write only the operation and explain why.

1. You drink $\frac{3}{8}$ liter in the morning and $\frac{1}{4}$ liter later.
   Operation: ______________ *Why?* ____
2. A $\frac{3}{4}$-yard ribbon is cut into pieces $\frac{1}{8}$ yard long.
   Operation: ______________ *Why?* ____
3. You use $\frac{2}{3}$ of a $\frac{3}{4}$-cup portion.
   Operation: ______________ *Why?* ____
4. You have $2\frac{1}{2}$ cups and use $\frac{3}{4}$ cup.
   Operation: ______________ *Why?* ____

Estimation: the error detector

Big idea

Before exact computation, decide what neighborhood the answer should live in.

Useful landmarks:

$$0, \quad \frac{1}{4}, \quad \frac{1}{2}, \quad \frac{3}{4}, \quad 1, \quad 2, \quad 5, \quad 10.$$

Examples:

- $\frac{7}{8} + \frac{5}{6}$ is almost $1 + 1$, so the answer should be a little less than 2.
- $\frac{1}{3} \times \frac{2}{5}$ is a fraction of a fraction, so the answer should be smaller than $\frac{1}{3}$ and smaller than $\frac{2}{5}$.
- $1 \div \frac{1}{8}$ asks how many eighths fit in 1, so the answer should be much bigger than 1.
- $7\frac{3}{5} \div 3\frac{1}{3}$ is roughly $7.6 \div 3.3$, so the answer should be a little above 2.

Your turn

No exact calculation. Give a rough range and explain.

1. $\frac{11}{12} + \frac{7}{8}$ is between __________ and __________.
   Because: ____

2. $\frac{4}{5} \times \frac{3}{5}$ is between __________ and __________.
   Because: ____

3. $2 \div \frac{1}{5}$ is between __________ and __________.
   Because: ____

4. $5\frac{1}{2} - 2\frac{3}{4}$ is between __________ and __________.
   Because: ____

Concept Lab 1: build the answer before calculating

For each problem, complete the four boxes. The arithmetic is last.

$\frac{3}{4} + \frac{1}{6}$

1. In words: ______

2. Picture I would draw: ______

3. Expected size: ______

4. Exact calculation: ______

$\frac{2}{3} \times \frac{3}{4}$

1. In words: ______

2. Picture I would draw: ______

3. Expected size: ______

4. Exact calculation: ______

$\frac{5}{6} \div \frac{1}{3}$

1. In words: ______

2. Picture I would draw: ______

3. Expected size: ______

4. Exact calculation: ______

Concept Lab 2: same numbers, different operation

The symbols $\frac{1}{2}$ and $\frac{1}{3}$ can appear in three very different questions.

Expression	Translation	What should happen?
$\frac{1}{2}+\frac{1}{3}$	combine one half and one third	answer larger than $\frac{1}{2}$
$\frac{1}{2}\times\frac{1}{3}$	one half of one third	answer smaller than both
$\frac{1}{2}\div\frac{1}{3}$	how many thirds fit in one half?	more than 1 group, but less than 2

Exact answers:

$$\frac{1}{2}+\frac{1}{3}=\frac{5}{6},\qquad \frac{1}{2}\times\frac{1}{3}=\frac{1}{6},\qquad \frac{1}{2}\div\frac{1}{3}=\frac{3}{2}=1\frac{1}{2}.$$

Big idea

The operation symbol changes the **story**. Before doing arithmetic, read the story.

Your turn

Use $\frac{3}{4}$ and $\frac{2}{5}$ to make three expressions: one addition, one multiplication, one division. For each, write a one-sentence story and predict whether the answer is below 1, near 1, or above 1.

Part VI: Word problems as translation practice

The word machine

A word problem is not "extra hard math." It is a translation task: ordinary language $\to$ mathematical structure $\to$ arithmetic.

Example 1: addition

A hiker walks $1\frac{3}{4}$ miles before lunch and $2\frac{2}{3}$ miles after lunch. How far altogether?

Meaning: combine two distances.
Operation: addition.
Estimate: about $2 + 3 = 5$ miles, but slightly less.
Exact: $1\frac{3}{4} + 2\frac{2}{3} = 4\frac{5}{12}$ miles.

Example 2: multiplication

A recipe uses $\frac{3}{4}$ cup of milk. You make $\frac{2}{3}$ of the recipe. How much milk?

Meaning: take two thirds *of* three fourths.
Operation: multiplication.
Exact: $\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$ cup.

Example 3: division

You have $2\frac{1}{4}$ yards of cord. Each piece must be $\frac{3}{8}$ yard long. How many pieces can you cut?

Meaning: how many $\frac{3}{8}$-yard groups fit in $2\frac{1}{4}$ yards?
Operation: division.
Exact: $2\frac{1}{4} \div \frac{3}{8} = 6$ pieces.

Word problems: identify first, compute second

Your turn

For each problem, write the operation and a sentence explaining why. Then calculate.

1. A tank is $\frac{3}{5}$ full. You use $\frac{1}{4}$ of the water that is currently in the tank. What fraction of the whole tank do you use?
   Operation: ______________ Why: ____
   Answer: ________________

2. You have $\frac{7}{8}$ pound of clay and divide it equally among 5 students. How much does each receive?
   Operation: ______________ Why: ____
   Answer: ________________

3. One board is $3\frac{1}{4}$ feet long and another is $1\frac{5}{6}$ feet long. What is their total length?
   Operation: ______________ Why: ____
   Answer: ________________

4. You have $4\frac{1}{2}$ liters of water. Bottles hold $\frac{3}{4}$ liter each. How many bottles can be filled?
   Operation: ______________ Why: ____
   Answer: ________________

Part VII: Become a fraction detective

Big idea

One of the fastest ways to learn is to explain **why a wrong answer is wrong**. This forces the rule to connect to meaning.

Your turn

A student wrote each answer below. Diagnose the mistake in words, then fix it.

1. $\frac{1}{2} + \frac{1}{3} = \frac{2}{5}$
   What went wrong? ______________________________
   Correct answer: __________

2. $\frac{2}{3} \times \frac{3}{4} = \frac{5}{6}$
   What went wrong? ______________________________
   Correct answer: __________

3. $1 \div \frac{1}{5} = \frac{1}{5}$
   What went wrong? ______________________________
   Correct answer: __________

4. $\frac{4}{5} \div \frac{4}{5} = \frac{16}{25}$
   What went wrong? ______________________________
   Correct answer: __________

5. $2\frac{1}{3} = \frac{5}{3}$
   What went wrong? ______________________________
   Correct answer: __________

Part VIII: Short computation bursts

Guide note

These are intentionally short. Stop after one page. Accuracy and explanation matter more than speed. For each row, the learner should verbally explain at least one problem.

Burst A: same denominators

1. $\frac{3}{8} + \frac{2}{8} =$
2. $\frac{7}{10} - \frac{4}{10} =$
3. $\frac{5}{12} + \frac{4}{12} =$
4. $\frac{11}{15} - \frac{2}{15} =$
5. $\frac{5}{9} + \frac{1}{9} =$
6. $\frac{13}{20} - \frac{7}{20} =$

Burst B: common denominators

1. $\frac{1}{2} + \frac{1}{4} =$
2. $\frac{2}{3} + \frac{1}{6} =$
3. $\frac{3}{4} - \frac{1}{8} =$
4. $\frac{2}{5} + \frac{3}{10} =$
5. $\frac{5}{6} - \frac{1}{4} =$
6. $\frac{3}{8} + \frac{5}{12} =$

Burst C: mixed numbers

1. $3 + 1\frac{3}{4} =$
2. $2\frac{1}{2} + 1\frac{1}{4} =$
3. $5\frac{2}{3} - 2\frac{1}{3} =$
4. $4\frac{1}{8} + 3\frac{3}{4} =$

Short computation bursts: multiplication

Burst D: fraction times fraction

1. $\frac{1}{2} \times \frac{2}{3} =$
2. $\frac{3}{5} \times \frac{1}{4} =$
3. $\frac{7}{8} \times \frac{2}{7} =$
4. $\frac{4}{9} \times \frac{3}{8} =$
5. $\frac{5}{6} \times \frac{3}{10} =$
6. $\frac{2}{5} \times \frac{5}{8} =$

Burst E: whole/mixed multiplication

1. $4 \times 1\frac{1}{2} =$
2. $6 \times 2\frac{1}{3} =$
3. $3 \times 2\frac{3}{4} =$
4. $5 \times 1\frac{2}{5} =$
5. $2\frac{1}{2} \times 4 =$
6. $1\frac{3}{4} \times 2 =$

Think before you calculate

Choose one multiplication problem whose answer is smaller than one factor, and explain why that is not a contradiction.

Short computation bursts: division

Burst F: meaning first

For each problem, write S for **sharing** or M for **measurement / how many fit** before solving.

1. $\frac{1}{2} \div 3 =$ S/M: ____
2. $1 \div \frac{1}{4} =$ S/M: ____
3. $\frac{3}{4} \div \frac{1}{8} =$ S/M: ____
4. $\frac{5}{6} \div 5 =$ S/M: ____
5. $\frac{4}{5} \div \frac{3}{10} =$ S/M: ____
6. $\frac{4}{5} \div \frac{4}{5} =$ S/M: ____

Burst G: mixed-number division

1. $2\frac{1}{4} \div \frac{3}{8} =$
2. $5\frac{1}{2} \div 2\frac{1}{4} =$
3. $4\frac{1}{2} \div \frac{3}{4} =$
4. $7\frac{3}{5} \div 3\frac{1}{3} =$

Think before you calculate

Which answer on this page should be exactly 1? Which should be greater than 4? Explain without recalculating.

Part IX: Mixed practice - read, predict, solve

For each problem, write a tiny symbol above the operation:

$+$ = combine $\quad$ $-$ = remove/compare $\quad$ $\times$ = of/scale $\quad$ $\div$ = share/how-many-fit

1. $\frac{2}{3} + \frac{5}{8} =$
2. $3\frac{1}{4} - 1\frac{5}{6} =$
3. $\frac{4}{7} \times \frac{21}{16} =$
4. $\frac{3}{4} \div \frac{2}{9} =$
5. $8 + 2\frac{5}{8} =$
6. $\frac{7}{12} + \frac{5}{18} =$
7. $\frac{5}{6} \times \frac{9}{20} =$
8. $1\frac{1}{2} \div \frac{3}{8} =$
9. $6 \times 1\frac{5}{6} =$
10. $\frac{11}{15} - \frac{2}{5} =$
11. $\frac{7}{9} \div 7 =$
12. $2\frac{2}{3} \times 1\frac{1}{8} =$

Think before you calculate

Pick any 3 problems. For each, write one sentence explaining what the expression means in ordinary English.

Concept check: no arithmetic allowed

Answer in words only.

1. Why can you add $\frac{3}{8} + \frac{2}{8}$ immediately, but not $\frac{3}{8} + \frac{2}{5}$?

---

2. Why does $1 \div \frac{1}{4}$ equal a number bigger than 1?

---

3. Why is $\frac{1}{3} \times \frac{1}{3}$ smaller than $\frac{1}{3}$?

---

4. What does a common denominator accomplish conceptually?

---

5. What does converting $2\frac{3}{4}$ to $\frac{11}{4}$ mean, rather than merely "doing a rule"?

---

6. Explain why $\frac{4}{5} \div \frac{4}{5} = 1$ without using the reciprocal algorithm.

The one-page fraction translator

If you see...	Say this to yourself
$\frac{a}{b}$	"a pieces, where each piece is one bth of the whole." It can also express the comparison "a to b."
$a : b$	"a of the first quantity for every b of the second." Name both quantities.
Reduce $\frac{a}{b}$	"Find a common bundle size that fits both counts; divide both by that same factor."
$\frac{a}{b} + \frac{c}{d}$	"Combine these amounts. Are the pieces the same size? If not, rename them with a common denominator."
$\frac{a}{b} - \frac{c}{d}$	"Remove or compare. I need same-size pieces before I subtract."
$\frac{a}{b} \times \frac{c}{d}$	"Take $\frac{a}{b}$ *of* $\frac{c}{d}$. This is scaling."
$\frac{a}{b} \div n$	"Share $\frac{a}{b}$ equally among n groups."
$\frac{a}{b} \div \frac{c}{d}$	"How many groups of size $\frac{c}{d}$ fit inside $\frac{a}{b}$?"
$a\frac{b}{c}$	"a whole units plus $\frac{b}{c}$."
Improper fraction	"Count how many denominator-sized pieces exist in all the wholes."
Reciprocal	"How many divisor-sized units per whole?"
Estimate	"What neighborhood should the answer live in?"

Big idea

Meaning first. Picture second. Estimate third. Arithmetic fourth.

Mastery challenge

For each problem, do four things: **translate, predict, solve, explain why the answer makes sense.**

1. $\frac{5}{6} + \frac{7}{12}$
2. $4\frac{3}{8} - 2\frac{5}{6}$
3. $\frac{3}{5} \times 1\frac{1}{4}$
4. $2\frac{2}{3} \div \frac{4}{9}$
5. $\frac{7}{10} \div \frac{14}{25}$
6. $8 \times 1\frac{7}{8}$
7. $1\frac{1}{2} + \frac{2}{3} + \frac{5}{12}$
8. A $3\frac{3}{4}$-mile trail is divided into sections $\frac{5}{8}$ mile long. How many section-lengths fit in the trail?
9. Reduce $\frac{84}{126}$ to lowest terms. Name the largest common bundle size you used.
10. A comparison is 300 to 9000. Write it as a reduced fraction and as a reduced ratio, then say the ratio in words.

Mastery is not “I remember the rule.”

Mastery is “I can explain what the symbols mean, predict the answer’s size, and choose a method that makes sense.”

Answer key and explanations

Core screenshot-style problems

1. $12 + 2\frac{3}{4} = 14\frac{3}{4}$.
2. $\frac{1}{2} + \frac{1}{2} = 1$ because two halves make one whole.
3. $7\frac{3}{5} \div 3\frac{1}{3} = 2\frac{7}{25}$; the estimate is a little above 2.
4. $\frac{1}{2} \div 3 = \frac{1}{6}$ because one half shared 3 ways makes sixths.
5. $\frac{4}{5} \div \frac{3}{10} = 2\frac{2}{3}$ because $\frac{4}{5} = \frac{8}{10}$ and $\frac{3}{10}$ fits $2\frac{2}{3}$ times.
6. $1 \div \frac{1}{4} = 4$ because four quarter-size groups fit in one whole.
7. $\frac{4}{5} \div \frac{4}{5} = 1$ because one copy of a number fits inside itself.
8. $6 \times 2\frac{1}{3} = 14$.
9. $\frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$.
10. $\frac{2}{5} \times \frac{5}{8} = \frac{1}{4}$.
11. $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$.

Selected earlier practice

Equivalent fractions: examples include $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12}$; $\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16}$; $\frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{8}{20}$.

Common denominators: $\frac{1}{2} + \frac{1}{5} \to 10$; $\frac{3}{4} + \frac{1}{6} \to 12$; $\frac{5}{8} + \frac{3}{10} \to 40$; $\frac{7}{12} - \frac{1}{6} \to 12$; $\frac{2}{9} + \frac{5}{6} \to 18$; $\frac{3}{14} + \frac{2}{7} \to 14$.

Addition/subtraction examples: $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$; $\frac{3}{4} + \frac{2}{5} = 1\frac{3}{20}$; $\frac{7}{8} - \frac{1}{3} = \frac{13}{24}$; $\frac{5}{6} - \frac{1}{4} = \frac{7}{12}$.

Mixed subtraction practice: $3\frac{1}{5} - 1\frac{3}{4} = 1\frac{9}{20}$; $6\frac{2}{3} - 2\frac{5}{6} = 3\frac{5}{6}$; $8\frac{1}{4} - 3\frac{7}{8} = 4\frac{3}{8}$.

Answer key: ratios and reduction

Fractions as ratios

$4 : 7 = \frac{4}{7}$; $3 : 5 = \frac{3}{5}$; $12 : 4 = \frac{12}{4} = 3$. The fraction notation expresses the first quantity divided by the second; ratio notation preserves the language "first to second."

For 4 green apples and 6 red apples (10 total):

- green:red $= 4 : 6 = 2 : 3$;
- red:green $= 6 : 4 = 3 : 2$;
- green:total $= 4 : 10 = 2 : 5$;
- fraction of all apples that are green $= \frac{4}{10} = \frac{2}{5}$.

Reducing by common bundle sizes

$\frac{8}{12} = \frac{2}{3}$ using bundles of 4; $\frac{15}{25} = \frac{3}{5}$ using bundles of 5; $\frac{14}{21} = \frac{2}{3}$ using bundles of 7.

Lowest terms practice: $\frac{18}{24} = \frac{3}{4}$; $\frac{21}{49} = \frac{3}{7}$; $\frac{36}{60} = \frac{3}{5}$; $\frac{45}{100} = \frac{9}{20}$.

Fraction form and ratio form

Original comparison	Reduced fraction	Reduced ratio
20 to 100	$\frac{1}{5}$	$1 : 5$
45 to 60	$\frac{3}{4}$	$3 : 4$
300 to 9000	$\frac{1}{30}$	$1 : 30$

Big idea

In the last example, both 300 and 9000 can be measured in 300-sized bundles: $300 = 1 \times 300$ and $9000 = 30 \times 300$. That is why the relationship becomes 1 to 30.

Answer key: short bursts

Burst A

$\frac{5}{8}$, $\frac{3}{10}$, $\frac{3}{4}$, $\frac{3}{5}$, $\frac{2}{3}$, $\frac{3}{10}$.

Burst B

$\frac{3}{4}$, $\frac{5}{6}$, $\frac{5}{8}$, $\frac{7}{10}$, $\frac{7}{12}$, $\frac{19}{24}$.

Burst C

$4\frac{3}{4}$, $3\frac{3}{4}$, $3\frac{1}{3}$, $7\frac{7}{8}$.

Burst D

$\frac{1}{3}$, $\frac{3}{20}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{4}$.

Burst E

6, 14, $8\frac{1}{4}$, 7, 10, $3\frac{1}{2}$.

Burst F

$\frac{1}{6}$, 4, 6, $\frac{1}{6}$, $2\frac{2}{3}$, 1.

Burst G

6, $2\frac{4}{9}$, 6, $2\frac{7}{25}$.

Mixed practice

1. $1\frac{7}{24}$
2. $1\frac{5}{12}$
3. $\frac{3}{4}$
4. $3\frac{3}{8}$
5. $10\frac{5}{8}$
6. $\frac{31}{36}$
7. $\frac{3}{8}$
8. 4
9. 11
10. $\frac{1}{3}$
11. $\frac{1}{9}$
12. 3

Answer key: mastery challenge

1. $\frac{5}{6}+\frac{7}{12}=\frac{10}{12}+\frac{7}{12}=\frac{17}{12}=1\frac{5}{12}$.

2. $4\frac{3}{8}-2\frac{5}{6}=\frac{35}{8}-\frac{17}{6}=\frac{105-68}{24}=\frac{37}{24}=1\frac{13}{24}$.

3. $\frac{3}{5}\times 1\frac{1}{4}=\frac{3}{5}\times\frac{5}{4}=\frac{3}{4}$.

4. $2\frac{2}{3}\div\frac{4}{9}=\frac{8}{3}\times\frac{9}{4}=6$.

5. $\frac{7}{10}\div\frac{14}{25}=\frac{7}{10}\times\frac{25}{14}=\frac{5}{4}=1\frac{1}{4}$.

6. $8\times 1\frac{7}{8}=8\times\frac{15}{8}=15$.

7. $1\frac{1}{2}+\frac{2}{3}+\frac{5}{12}=\frac{18}{12}+\frac{8}{12}+\frac{5}{12}=\frac{31}{12}=2\frac{7}{12}$.

8. $3\frac{3}{4}\div\frac{5}{8}=\frac{15}{4}\times\frac{8}{5}=6$ section-lengths.

9. $\frac{84}{126}=\frac{2}{3}$; the GCF is 42, so $84=2\times 42$ and $126=3\times 42$.

10. $\frac{300}{9000}=\frac{1}{30}$ and $300:9000=1:30$. In words: “1 unit of the first quantity for every 30 units of the second.”

Guide note

If the learner can explain **why** the answers above make sense but still makes small arithmetic slips, that is useful information. It means the conceptual layer is strengthening, and computation can then be trained in short, low-friction bursts rather than by overwhelming the learner with long worksheets.

End of workbook

Read the symbols. Tell the story. Then calculate.